

## The Tachyon Potential in Open Neveu-Schwarz String Field Theory

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A classical action for open superstring field theory has been proposed which does not suffer from contact term problems. After generalizing this action to include the non-GSO projected states of the Neveu-Schwarz string, the pure tachyon contribution to the tachyon potential is explicitly computed. The potential has a minimum of  $V = -\frac{1}{32g^2}$  which is 60% of the predicted exact minimum of  $V = -\frac{1}{2\pi^2 g^2}$  from D-brane arguments.

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## 1. Introduction

The Neveu-Schwarz (NS) sector of the non-GSO projected superstring has recently been reconsidered as part of a sensible physical theory [1][2]. Although this sector contains a tachyon, there have been proposals for removing the undesired properties of the the tachyon by assuming a tachyon potential which is bounded from below.

The most efficient method for computing the tachyon potential uses open string field theory[3] [4] [5], however the cubic action of [6] for open superstring field theory contains contact term problems which spoil gauge invariance[7]. Recently, a new action for open superstring field theory has been constructed [8] which does not suffer from contact term problems. This action resembles a Wess-Zumino-Witten action and can be naturally obtained by embedding the N=1 description of the superstring into an N=2 string [9].

In this paper, the pure tachyon contribution to the tachyon potential will be explicitly computed using this new action. The pure tachyon contribution is

$$V(T) = -\frac{1}{4g^2}T^2 + \frac{1}{2g^2}T^4, \quad (1.1)$$

which has a minimum of  $V(T_0) = -\frac{1}{32g^2}$  when  $T_0 = \pm\frac{1}{2}$ . This value of the minimum is 60% of the predicted exact minimum of  $V(T_0) = -\frac{1}{2\pi^2g^2}$  using D-brane arguments<sup>2</sup> where the mass of the brane-antibrane is  $\frac{1}{2\pi^2g^2}$  [4][10]. It would be interesting to check if the remaining 40% comes from including contributions to the effective tachyon potential from non-tachyon fields, as was found for the bosonic string tachyon potential in [5].

## 2. Neveu-Schwarz String Field Theory Action

Using the superstring field theory action of [8], the GSO-projected NS contribution is given by

$$S = \frac{1}{2g^2}Tr\langle(e^{-\Phi}Qe^{\Phi})(e^{-\Phi}\eta_0e^{\Phi}) - \int_0^1 dt(e^{-t\Phi}\partial_t e^{t\Phi})\{e^{-t\Phi}Qe^{t\Phi}, e^{-t\Phi}\eta_0e^{t\Phi}\}\rangle \quad (2.1)$$

where  $\eta_0 = \oint dz\eta(z)$  is defined by fermionizing the super-reparameterization ghosts as[11]  $\gamma = \eta e^{\phi}$  and  $\beta = \partial\xi e^{-\phi}$ ,

$$Q = \oint dz[c(T_{matter} - \eta\partial\xi - \frac{1}{2}\partial\phi\partial\phi - \partial^2\phi - b\partial c) + \eta e^{\phi}G_{matter} - \eta\partial\eta e^{2\phi}b],$$

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<sup>2</sup> In the original version of this paper, the mass of the brane-antibrane was incorrectly stated to be  $\frac{1}{\pi^2g^2}$ . This value of the mass is only correct if one doubles the number of states in the string field theory action to allow for strings which end on the brane or antibrane [10] .

and  $\langle \rangle$  signifies the two-dimensional correlation function in the “large” RNS Hilbert space [11] where  $\langle \xi c \partial c \partial^2 c e^{-2\phi} \rangle = 2$ . The normalization of (2.1) has been fixed by requiring that the quadratic Yang-Mills contribution to the action is  $S = -\frac{1}{4g^2} \text{Tr} \int d^{10}x F_{mn} F^{mn}$ , which is the correct sign for the  $(- + \dots +)$  metric that is being used. String fields are multiplied using the midpoint interaction of [12] and  $\Phi$  is related to the NS string field  $V$  of [6] by  $\Phi = \xi_0 V$  or  $V = \eta_0 \Phi$ .

In the GSO-projected sector, the NS string field  $\Phi$  is bosonic. Since the unprojected NS states are fermionic with respect to the projected NS states, it will be convenient to define  $\hat{\Phi} = \Phi_B \times I + \Phi_F \times \sigma_1$  where  $\Phi_B$  described the projected states,  $\Phi_F$  describes the unprojected states,  $I$  is the  $2 \times 2$  identity matrix, and  $(\sigma_1, \sigma_2, \sigma_3)$  are the Pauli matrices [4]. Furthermore, it will be convenient to define

$$\hat{Q} \equiv Q \times \sigma_3, \quad \hat{\eta}_0 \equiv \eta_0 \times \sigma_3,$$

which satisfy  $\hat{Q}(\hat{\Phi}_1 \hat{\Phi}_2) = (\hat{Q} \hat{\Phi}_1) \hat{\Phi}_2 + \hat{\Phi}_1 (\hat{Q} \hat{\Phi}_2)$  and  $\hat{\eta}_0(\hat{\Phi}_1 \hat{\Phi}_2) = (\hat{\eta}_0 \hat{\Phi}_1) \hat{\Phi}_2 + \hat{\Phi}_1 (\hat{\eta}_0 \hat{\Phi}_2)$ .

The complete non-GSO projected NS string field theory action is defined by

$$S = \frac{1}{4g^2} \text{Tr} \langle (e^{-\hat{\Phi}} \hat{Q} e^{\hat{\Phi}}) (e^{-\hat{\Phi}} \hat{\eta}_0 e^{\hat{\Phi}}) - \int_0^1 dt (e^{-t\hat{\Phi}} \partial_t e^{t\hat{\Phi}}) \{ e^{-t\hat{\Phi}} \hat{Q} e^{t\hat{\Phi}}, e^{-t\hat{\Phi}} \hat{\eta}_0 e^{t\hat{\Phi}} \} \rangle \quad (2.2)$$

where the trace is over the  $2 \times 2$  matrices as well as the Chan-Paton matrices.

One can check that (2.2) is invariant under the WZW-like gauge transformation

$$\delta e^{\hat{\Phi}} = (\hat{Q} \hat{\Omega}) e^{\hat{\Phi}} + e^{\hat{\Phi}} (\hat{\eta}_0 \hat{\Omega}') \quad (2.3)$$

where  $\hat{\Omega}$  and  $\hat{\Omega}'$  are string fields of the form  $\hat{\Omega} = \Omega_F \times \sigma_3 + i\Omega_B \times \sigma_2$  with  $\Omega_F$  being fermionic and projected while  $\Omega_B$  is bosonic and unprojected. One subtle point in proving this gauge invariance is that  $\langle \hat{\Phi}_1 \hat{\Phi}_2 \rangle = \langle \hat{\Phi}_2 \hat{\Phi}_1 \rangle$  since when  $A$  and  $B$  are unprojected states,  $\langle AB \rangle = \mp \langle BA \rangle$  where the minus sign is if they are bosons and the plus sign is if they are fermions. This reversal of the usual statistics comes from square-root factors produced by the  $\frac{1}{2}$ -integer conformal weight of unprojected NS states. Note that a similar subtlety occurs with unprojected states using the action of [6].

### 3. Computation of Tachyon Potential

Expanding the action of (2.2) in powers of  $\widehat{\Phi}$ , one obtains

$$S = \frac{1}{2g^2} Tr \langle \frac{1}{2}(\widehat{Q}\widehat{\Phi})(\widehat{\eta}_0\widehat{\Phi}) - \frac{1}{6}\widehat{\Phi}\{\widehat{Q}\widehat{\Phi}, \widehat{\eta}_0\widehat{\Phi}\} - \frac{1}{24}[\widehat{\Phi}, \widehat{Q}\widehat{\Phi}][\widehat{\Phi}, \widehat{\eta}_0\widehat{\Phi}] + \dots \rangle. \quad (3.1)$$

To compute the term with  $N$   $\widehat{\Phi}$ 's, one uses the map

$$w(z) = \left( \frac{1 - iz}{1 + iz} \right)^{\frac{2}{N}}$$

from the disc to a  $2\pi/N$  wedge of the complex plane. Rotating this map by a factor  $e^{\frac{2\pi i}{N}}$  allows each successive string field to get mapped to a different  $2\pi/N$  wedge. The center of the  $J^{th}$  disc gets mapped to the point  $e^{\frac{2\pi i(J-1)}{N}}$  and, to obtain an  $SL(2, \mathbb{R})$ -invariant expression, the  $J^{th}$  string field gets multiplied by a factor  $(e^{\frac{2\pi i(J-1)}{N}} \frac{4}{Ni})^h$  where  $h$  is the conformal weight of the string field and  $\frac{4}{Ni}$  is  $\frac{dw}{dz}|_{z=0}$  [13].

The tachyon field  $T(x)$  appears in the string field  $\widehat{\Phi}$  as

$$\widehat{\Phi} = i\xi c e^{-\phi} T(x) \times \sigma_1 \quad (3.2)$$

where the factor of  $i$  is needed to get the right sign for the kinetic term. One can easily compute that at zero momentum,

$$\widehat{Q}\widehat{\Phi} = T(\frac{1}{2}c\partial c\xi e^{-\phi} + \eta e^{\phi}) \times \sigma_2, \quad \widehat{\eta}_0\widehat{\Phi} = -Tce^{-\phi} \times \sigma_2. \quad (3.3)$$

Since  $\langle \xi c \partial c \partial^2 c e^{-2\phi} \rangle = 2$ , the only pure tachyon contribution to the action of (2.2) comes from the quadratic and quartic terms of (3.1). The quadratic contribution to the action (which is minus the tachyon potential) is given by

$$-V_2 = -\frac{1}{2g^2} T^2 \langle (\frac{1}{2}c\partial c\xi e^{-\phi}(1))(ce^{-\phi}(-1)) \rangle (-2i)^{-\frac{1}{2}} (2i)^{-\frac{1}{2}} = \frac{1}{4g^2} T^2. \quad (3.4)$$

The quartic contribution to the action is given by

$$\begin{aligned} -V_4 = & \frac{T^4}{24g^2} (-i)^{-\frac{1}{2}} (1)^{-\frac{1}{2}} (i)^{-\frac{1}{2}} (-1)^{-\frac{1}{2}} \langle (\xi c e^{-\phi}(1))(\eta e^{\phi}(i))(\xi c e^{-\phi}(-1))(ce^{-\phi}(-i)) \\ & + (\eta e^{\phi}(1))(\xi c e^{-\phi}(i))(\xi c e^{-\phi}(-1))(ce^{-\phi}(-i)) + (\xi c e^{-\phi}(1))(\eta e^{\phi}(i))(ce^{-\phi}(-1))(\xi c e^{-\phi}(-i)) \\ & + (\eta e^{\phi}(1))(\xi c e^{-\phi}(i))(ce^{-\phi}(-1))(\xi c e^{-\phi}(-i)) \rangle = -\frac{T^4}{2g^2}. \end{aligned} \quad (3.5)$$

So  $V(T) = V_2 + V_4 = -\frac{1}{4g^2} T^2 + \frac{1}{2g^2} T^4$  which has a minimum of  $V(T_0) = -\frac{1}{32g^2}$  when  $T_0 = \pm \frac{1}{2}$ .

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## References

- [1] A. Sen, *Stable Non-BPS States in String Theory* JHEP 9806 (1998) 007, hep-th/9803194;  
A. Sen, *Tachyon Condensation on the Brane Antibrane System* JHEP 9808 (1998) 012, hep-th/9805170.
- [2] O. Bergman and M.R. Gaberdiel, *A Non-Supersymmetric Open String Theory and S-Duality*, Nucl. Phys. B499 (1997) 183, hep-th/9701137;  
T. Yoneya, *Spontaneously Broken Space-Time Supersymmetry in Open String Theory without GSO Projection*, hep-th/9912255.
- [3] V.A. Kostelecky and S. Samuel, *The Static Tachyon Potential in the Open Bosonic String Theory*, Phys. Lett. B207 (1988) 169.
- [4] A. Sen, *Universality of the Tachyon Potential*, hep-th/9911116.
- [5] A. Sen and B. Zwiebach, *Tachyon Condensation in String Field Theory*, hep-th/9912249.
- [6] E. Witten, *Interacting Field Theory of Open Superstrings*, Nucl. Phys. B276 (1986) 291.
- [7] C. Wendt, *Scattering Amplitudes and Contact Interactions in Witten's Superstring Field Theory*, Nucl. Phys. B314 (1989) 209;  
J. Greensite and F.R. Klinkhamer, *Superstring Amplitudes and Contact Interactions*, Nucl. Phys. B304 (1988) 108.
- [8] N. Berkovits, *Super-Poincare Invariant Superstring Field Theory*, Nucl. Phys. B450 (1995) 90, hep-th 9503099;  
N. Berkovits, *A New Approach to Superstring Field Theory*, proceedings to the 32<sup>nd</sup> International Symposium Ahrenshoop on the Theory of Elementary Particles, Fortschritte der Physik (Progress of Physics) 48 (2000) 31, hep-th/9912121;  
N. Berkovits and C.T. Echevarria, *Four-Point Amplitude from Open Superstring Field Theory*, hep-th/9912120.
- [9] N. Berkovits and C. Vafa,  *$N=4$  Topological Strings*, Nucl. Phys. B433 (1995) 123, hep-th/9407190.
- [10] A. Sen, private communication.
- [11] D. Friedan, E. Martinec, and S. Shenker, *Conformal Invariance, Supersymmetry, and String Theory*, Nucl. Phys. B271 (1986) 93.
- [12] E. Witten, *Noncommutative Geometry and String Field Theory*, Nucl. Phys. B268 (1986) 253.
- [13] E. Cremmer, A. Schwimmer and C. Thorn, *The Vertex Function in Witten's Formulation of String Field Theory*, Phys. Lett. B179 (1986) 57;  
D.J. Gross and A. Jevicki, *Operator Formulation of Interacting String Field Theory*, Nucl. Phys. B283 (1987) 1;  
A. LeClair, M.E. Peskin and C.R. Preitschopf, *String Field Theory on the Conformal Plane (I)*, Nucl. Phys. B317 (1989) 411.